Use of thermodynamic entropy as thermal charge

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1. Introduction

Thermodynamics is considered a very difficult subject, expressed with so many letters and differentials. It becomes much simpler if one accepts the circuit thinking of electrical engineering. It is the purpose of this article to develop systematically the circuit and network thinking in thermodynamics and to go on from there. Instead of electric circuit elements, we use the Bondgraph (BG) formalism and we develop the BG symbols as needed. A pre-knowledge of BG is helpful, but not necessary to understand the definition of entropy in this article.

Here we shall develop the circuit thinking for thermodynamics and built the formalism from there. Circuit thinking was firstly developed in electrical engineering and for thermodynamics only slight modifications are required, as we shall develop.

The main point is to consider entropy as a substance, indeed as thermal charge. As such it has many similarities with electric charge, but also some differences. Entropy flow is connected to the transfer of heat power by the so-called Carnot’s equation

\[ \dot{Q} = T \dot{S} \quad \text{or} \quad \dot{S} = \frac{\dot{Q}}{T} \]

To repeat, entropy flow is the more familiar heat flow divided by absolute temperature.

Entropy flow is very similar to the transmission of electric power, or simply electric conduction, which has the equation

\[ \dot{E} = u \dot{q} = u i \]

With entropy as thermal charge, one can built circuits of thermodynamic installations, a little like electric circuits, but with slightly different properties. Here it is convenient to use the BG formalism and to show the difference to electric circuits by special BG elements. We will describe them, when we built the thermodynamic circuits.

2. Properties of entropy

Contrary to electric charge, which is always conserved is best to imagine entropy as a kind of substance, connected to heat flow by Carnot’s equation, and produced by all kinds of frictions or irreversibilities. Under friction we understand not only mechanical and electric friction, but also heat conduction under finite temperature difference.

Further mechanisms to generate entropy are unopposed mixing of fluids, and also the resistance in chemical reactions. They exist but are in practice not important enough to justify treatment here. We use the Bondgraph (BG) formalism here, where all frictions are R-elements. To indicate properly the entropy generation, the R-element becomes saddled with a source element, becoming thus an RS-element.

Important is that the single R-element is a power sink, but does not exist as such. It becomes a power conserving RS-element: all absorbed power in the R-part appears again as heat, or as entropy times absolute temperature. This is illustrated by Fig. 1, with the simple R-element at top and the power conserving RS-element at bottom:
Fig. 1: Including thermics, the R element becomes an RS element.

Let us repeat: a simple R-element does not exist and becomes a power conserving RS-element by including thermal effects in our BG.

3. Thermodynamic circuits

Fig. 2: Top: simple electric circuit with one resistor and two capacitors. Bottom: BG of this circuit.

One can built circuits for simple electric installations as on Fig. 2 with a resistor between two capacitors. As long as the voltages in the capacitors are different, there will flow a current through the resistor and we have
dissipation. It will appear in the thermal domain, as new entropy times absolute temperature as indicated below. Dissipation is a power flow that is always positive out from the resistor, even if the voltage changes sign. This is well shown by the equation

\[ I = Ru \quad \text{and} \quad \text{Dis} = \frac{u^2}{R} \]

where one sees that the dissipation is always positive (even if the voltage \( u \) is negative).

**Fig. 3:** BG for thermal conduction. Here the new entropy is injected to the C-element at lower temperature.

Thermal conduction is quite similar to electric conduction, with the dissipation in a thermal RS-element. Only its dissipation is thrown as entropy flow into the C-element having the lower temperature on the right of Fig. 3. Again, thermal dissipation is always positive. These electric dissipation and heat conduction are the two main mechanisms of generating entropy. If it is not clear which C-element is at lower temperature, a switch element \( \text{SW} \) can be interposed between the dissipating RS-field and both 0-junctions near to the C-elements. Such an element is quite easy to realize on a computer. It is notable that entropy has a semi-conservation property: it can never be destroyed, but is generated “out of nothing” by all kind of irreversibilities. To remove entropy from our field of interest, it must be transported away. Ultimately it must be disposed in stellar space around the earth.

**Picture of Carnot cycle**

**Fig. 4:** BG of the famous Carnot cycle, where two adiabatics are interposed between the entropy spending and absorbing isothermals, one hot, the other cold.
The celebrated cycle of Carnot (1824) is represented in our formalism as follows: at left, we find a continuous flow source, driving a modulated transformer MTF, which converts the rotation into alternating movement in a cylinder. The working gas is contained in a 2-port C-field -C-. The switch SW has 3 positions:

1. connecting to the high temperature in SE1;
2. connecting to the low temperature in SE2;
3. connecting to nothing, the so called adiabatics of Carnot.

As Thoma has explained in [THOMA 2009], to interpose the 2 adiabatics between the isothermals was a great invention by Carnot. It avoids thermal short circuit when switching from high temperature to low temperature and vice versa. In practice, the Carnot cycle is not used because too much of the power gained in the expansion is needed to drive the compression. In other words, the power ratio, also called work ratio, is too small, especially if the working medium is an ideal gas. Therefore, historically one used a two phases working medium, water and steam, where the feeding pump with water used only negligible power. This is called the steam engine, with its heavy boiler, which dominated the world. Later, it was superseded by the more efficient internal combustion engine with its burning of gas inside the working cylinders.

**Origins of entropy and heat**

All tables of hot matter or water and steam give only incremental values, which are from a certain reference. In a water-steam mixture, one assigns the origin to water just at freezing as the most stable state and counts the values from there. Entropy is given only in incremental values by the formula

\[
S_1 - S_2 = \int \frac{dQ}{T} \approx \int \frac{dQ_1}{T_1} - \int \frac{dQ_2}{T_2}
\]

There is also the Nernst theorem which says that at zero temperature the entropy content of all matter is zero. We hold that this is only a theoretical law, and that entropy content, especially at low temperature, is unknown: we don’t know the modifications of structure at cryogenic temperatures.

**4. Conclusion**

We care at the end of our trip through classical thermodynamics, which is simple and beautiful, if we take entropy as something fundamental, as thermal charge. It has special conservation properties: it can be generated “out of nothing” by frictions but it can never be destroyed and has ultimately to be disposed of in the stellar space around the earth. The important point is that all tables are giving only increments and leave the question of absolute entropy undecided. It would be good to give more thought to the absolute values of entropy.

**References**

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